Solutions for Recitation 12/8

1. stated: $p_0 = 0.3$

a) $H_0: p = 0.3$ (this is the null hypothesis and we usually set $p = p_0$)

$H_A: p > 0.3$ (this is the alternative hypothesis and we set $p > p_0$ b/c the hump in the graph is on the right)

In hypothesis testing, we always start with 2 hypothesis. H_0 is the simple (b/c we set p equal to a specific value) case, and H_A is what we are testing (% of beverage orders for Coke is greater than 30%).

b) α is called the Type I error. This error comes from rejecting the null hypothesis (H_0) even though it is true. We read α from the graph. α = value on the y-axis corresponding to $p = 0.3$ (x-axis) on the power function curve.

$\alpha = 0.2$

c) A Type II error is the case when: we fail to reject H_0 even though H_A (alternative hypothesis) is true.

We will go over (c)(d)(e) in recitation.

2. stated: both tests have the same α, and H_0 and H_A

a) Since both tests have the same α, then α must equal the point where both curves intersect.

$\alpha = 0.05$ and $p_0 = 0.7$ (point of intersection)

therefore $H_0: p = 0.7$

$H_A: p < 0.7$ (hump on left)

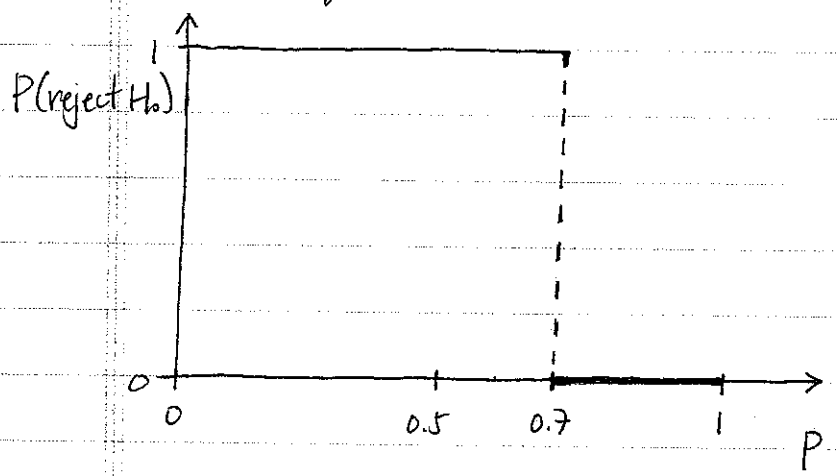
The test with the steeper slope is better since it is closer to the ideal power curve.

b) • $H_A: p < 0.7$ and $H_A: p > 0.7$ are both one sided tests

• $H_A: p \neq 0.7$ is a two sided test

So we have $H_A: p < 0.7$, hence a one sided test.

c) Ideal power curve:



3. stated: both tests have same α, and same H_0 and H_A

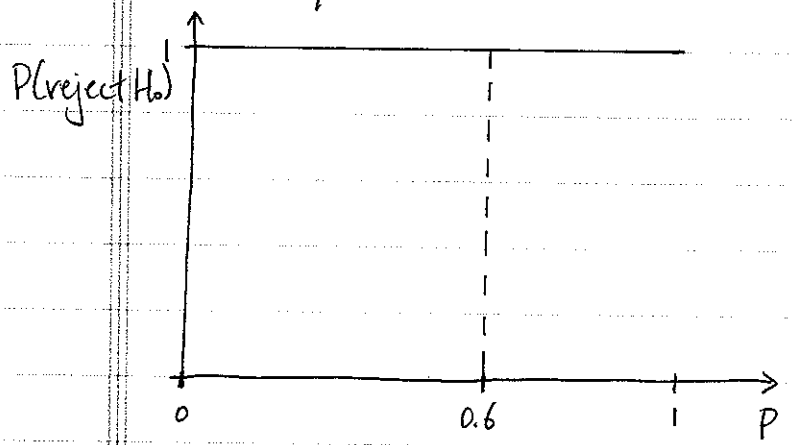
a) $\alpha = 0.1$, $p_0 = 0.6$ (point of intersection)

$H_0: p = 0.6$

$H_A: p \neq 0.6$ (b/c there are humps on left and right)

b) $\alpha = 0.1$

c) Ideal power curve:



d) The better test has the steeper slope (closer to the ideal power curve)

e) Let A = worse test of the two.

B = better test of the two for simplicity.

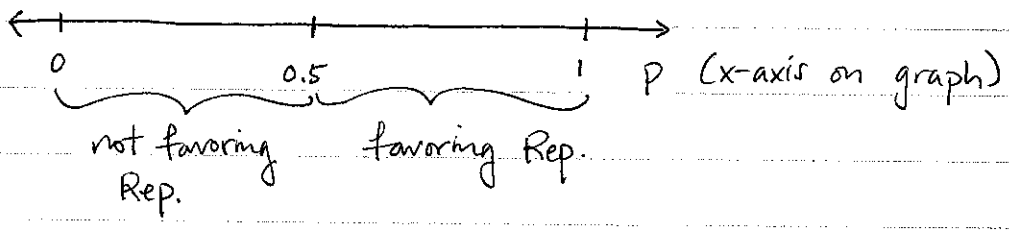
We look at the value on the y-axis that corresponds to $p = 0.55$ (x-axis) on both curves A and B.

For A: $P(\text{reject } H_0) = 0.4$

For B: $P(\text{reject } H_0) = 0.9$

4. stated: · p = % favoring republican
· n = 800 (not used)
· "null hypothesis that the Republican ... leading in votes"
· "alternative hypothesis is the Republican ... not leading in votes"

a) H_0 : $p > 0.5$ (As stated in the question)
H_A : $p < 0.5$



b) $\alpha = 0.4$ (corresponds to $p = 0.5$ on curve)
(corresponding y-axis value) favor Rep. ↓

c) Democrats, b/c if $p = 0.5$, there is a 40% chance to reject H_0.

d) P(reject H_0) = 0.95 (corresponds to $p = 0.45$ on curve)

e) We would make the curve symmetric such that if $p = 0.5$, there is a 50% chance to reject H_0 (fair to both parties).

5. stated: · n = 400
· $\alpha = 0.2$
· out of 400, there are 150 wine orders

a) $\hat{p}$ = proportion of interest in sample = $\frac{150}{400} = 0.375$

b) $p_0 = 30\% = 0.3$ so:

H_0 : $p = 0.3$ (stated in question)
H_A : $p > 0.3$

in book

c) $SD(p_0) = SD(\hat{p}) = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.3(0.7)}{400}} = 0.023$

d) $z = \frac{\hat{p} - p_0}{SD(p_0)} = \frac{0.375 - 0.3}{0.023} = 3.273$

e) p-value $= P(z > 3.273) = 0.000532$

f) if: p-value $> \alpha \Rightarrow$ do not reject H_0
p-value $< \alpha \Rightarrow$ reject H_0

Here p-value $= 0.000532$, $\alpha = 0.2$ so we reject H_0.
$\Rightarrow$ conclude "wine orders are greater than 30%"

g) do this in recitation

6. stated:
- $n = 400$
- $\alpha = 0.2$
- 90 wine orders

a) $\hat{p} = \frac{90}{400} = 0.225$

b) $p_0 = 0.3$ so
$H_0: p = 0.3$
$H_A: p \neq 0.3$ (as stated in question)

c) $SD(p_0) = SD(\hat{p}) = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.3(0.7)}{400}} = 0.023$

d) $z = \frac{\hat{p} - p_0}{SD(p_0)} = \frac{0.225 - 0.3}{0.023} = -3.273$

absolute

e) p-value $= 2 \times P(Z > |-3.273|)$

$= 2 \times P(Z > 3.273)$

$= 2 \times 0.000532$

$= 0.00106$

f) p-value $= 0.00106$, $\alpha = 0.2$

p-value $< \alpha$ so we reject H_0 $\Rightarrow$ conclude "wine orders is at other than 30%"

g) do this in recitation